Condition for amplitude of resonance:

In case of forced vibration the general solution for displacement at any instant is given by

$$y = ae^{-bt}\sin(\omega t-\alpha) + \frac{F}{\sqrt{\mu^2p^2+(k-mp^2)^2}}\sin(pt-\alpha) \rightarrow (1)$$

If the viscosity of the medius is less then the amplitude $a = \frac{F}{\sqrt{\mu^2p^2+(k-mp^2)^2}} \longrightarrow (2)$

will be maximum.

It is maximum, when the denominator is minimum. This is possible when $(k-mp^2)=0$.

$$\Rightarrow mp^2 = k$$

$$p = \sqrt{\frac{k}{m}}$$

$$\boxed{p = \omega} \longrightarrow (3)$$

It shows that amplitude will be maximum, when the natural frequency is equal to forced frequency. As this condition the oscillation is said to be under resonance.

Now, we can write the condition for amplitude of resonance as.

$$a = \frac{F}{\sqrt{\mu^2p^2+\cancel{(k-mp^2)^2}}_{0}}$$

$$a = \frac{F}{\sqrt{\mu^2p^2}}$$

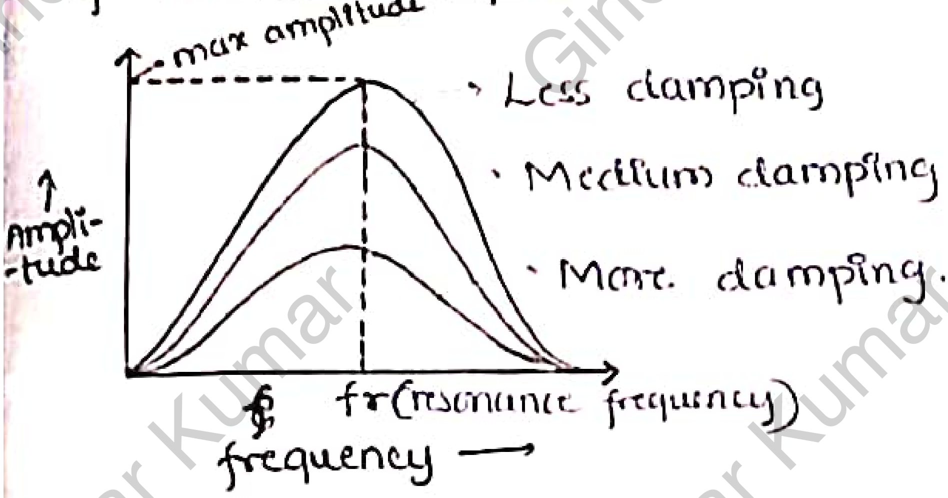
$$a = \frac{F}{\mu^2p^2}$$

$$\boxed{a = \frac{F}{\mu}\sqrt{\frac{m}{k}}} \longrightarrow (4)$$

Eq^n (4) represent the condition for amplitude of resonance.

Sharpness of Resonance:

Sharpness of Resonance means the fall in the amplitude with change in frequency on both side of maximum amplitude



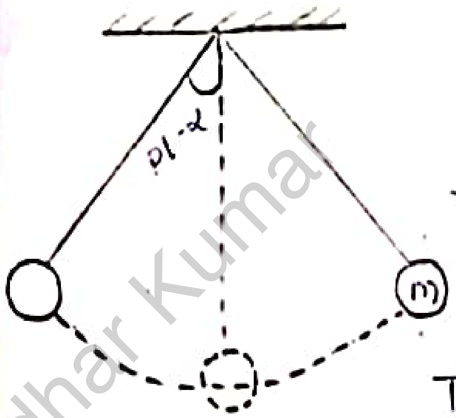
Let us consider particular solution of forced vibratition.

$$y = \frac{F}{\sqrt{\mu^2p^2 + (k-mp^2)^2}} \sin(pt-\alpha) \longrightarrow (1)$$

Differentiate eq^n (1) w.r.t time t.

$$\frac{dy}{dt} = \frac{FP}{\sqrt{\mu^2p^2+(k-mp^2)^2}} \cos(pt-\alpha) \longrightarrow (2)$$

The term $\frac{dy}{dt}$ represents the velocity and it is maximum, when $\cos(pt-\alpha)$ is equal to 1, it is possible, when particle just crosses the mean position.



$$\therefore \frac{dy}{dt} = \frac{FP}{\sqrt{\mu^2p^2+(k-mp^2)^2}} \times (1)$$

$$V_{max} = \frac{dy}{dt} = \frac{FP}{\sqrt{\mu^2p^2+(k-mp^2)^2}} \longrightarrow (3)$$

The K.E is maximum when the Particle just crosses the mean position it is given by

$$KE_{max} = \frac{1}{2} m V_{max}^2$$

$$KE_{max} = \frac{1}{2} m \left[\frac{F^2 p^2}{\mu^2 p^2 + (k-mp^2)^2}\right] \quad \text{---} (4) \quad (10)$$

The mean square of driving force per unit mass is given by $= \frac{\frac{0+F^2}{2}}{m} = \frac{F^2}{2m}$ —— (5)

The KE per unit force is given by Dividing eq^n (4) by (5) and it is called as Response (R)

$$R = \frac{m^2 p^2}{\mu^2 p^2 + (k-mp^2)^2}$$

$$R = \frac{p^2}{\frac{\mu^2 p^2}{m^2} + \left(\frac{k}{m} - p^2\right)^2} \quad \text{———} (6)$$

The natural frequency in absence of damping is $\sqrt{\frac{k}{m}}$, therefore the term $\left[\frac{k}{m} - p^2\right]$ in the denominator represent the extent to which the natural frequency differs from applied frequency

When, $\frac{k}{m} = p^2$ the natural frequency coinside with forced frequency.

Now, eq^n (6) becomes

$$R = \frac{p^2}{\frac{\mu^2 p^2}{m^2} + (0)}$$

$$R = \frac{\cancel{p^2}}{\frac{\mu^2 \cancel{p^2}}{m^2}}$$

$$\boxed{R = \left(\frac{m}{\mu}\right)^2} \quad \text{———} (7)$$

$$\boxed{R \propto \frac{1}{\mu}} \quad \text{———} (8)$$

μ = damping force constant